ROLL NO._____   **Class: XII (2019-20)**

Time : 3hrs   **MATHEMATICS (041)**   M.M. : 80

GENERAL INSTRUCTIONS

(i)   All questions are compulsory
(ii)   The question paper consists of 36 questions divided into four sections A, B, C and D. Section A comprises of 20 questions of one mark each. Section B comprises of 6 questions of two marks each. Section C comprises of 6 questions of four marks each and Section D comprises of 4 questions of six marks each.
(iii)   Use of calculators is not permitted.

SECTION –A

1. If $A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $BA = (b_{ij})$. Find $b_{21} + b_{32}$.

2. Is $f: \mathbb{N} \to \mathbb{N}$ given by $f(x) = x^2$ one-one? Justify your statement.

3. Express in simplest form: $\tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}$; $0 < x < \pi$.

4. Total cost C(x) in ₹ associated with the production of x units of an item is given by

   $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the Marginal cost M(x) when 17 units are produced, where Marginal cost is the instantaneous rate of change of total cost.

5. For what values of k, the function $f(x) = \begin{cases} kx + 1 & ; & x \leq \pi \\ \cos x & ; & x > \pi \end{cases}$ is continuous at $x = \pi$.

6. Find the value of $x - y$, if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

7. Differentiate $\cot(5x^0)$ w.r.t. x

8. What is the principle value of $\tan^{-1}\left(\tan \frac{2\pi}{3}\right)$.

9. Using differentials, Find the approximate value of $\sqrt{0.06}$

10. If A and B are two invertible matrices of order 3, $|A| = 2$ and $|(AB)^{-1}| = -\frac{1}{6}$. Find $|B|$.

11. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$; $xy < 1$. Write the value of $x + y + xy$.

12. If $y = 5e^{7x} + 6e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$.

13. If $A = \begin{pmatrix} 1 & -2 \\ 4 & 3 \end{pmatrix}$ & $B = \begin{pmatrix} 0 & 1 \\ 2 & 4 \end{pmatrix}$. Find $(2A + 5B)^T$.

14. Evaluate: $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

15. Write the number of all possible matrices of order 2X2 with each entry 1, 2 or 3.
16. Differentiate $\sin^2(x^2)$ w.r.t. x^2
17. If A is a square matrix of order 2 and $|A| = 4$, then find $|2.AA'|$, where A' is the transpose of matrix A.
18. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x^2 - 5$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = \frac{x}{x^2+1}$. Find gof.
19. For what values of k, the system of linear equations:

$$x + y + z = 2 \; ; \; 2x + y - z = 3 \; ; \; 3x + 2y + kz = 4 \quad 2x+y-z=3 \; ; \; 3x+2y+kz=4$$

20. If $y = x\,|x|$, find $\frac{dy}{dx}$ for $x < 0$.

SECTION –B

21. The volume of cube is increasing at the rate of 8cm³/sec. How fast is the surface area increasing when the length of an edge is 12cm?

22. Solve the equation: $\begin{vmatrix} x+1 & 2 & 3 \\ 3 & x+2 & 1 \\ 1 & 2 & x+3 \end{vmatrix} = 0$.

23. Discuss the continuity of the function $f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$, where $x \neq 0$ and $f(0) = 0$ at $x = 0$.

24. A book store has 20 Mathematics books, 15 Physics books and 12 Chemistry books. Their selling prices are ₹ 300, ₹ 320 and ₹ 340 each respectively. Find the total amount the store will receive from selling all the items.

25. Show that $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) = x - \frac{\pi}{4} \; ; \; \frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$.

26. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units/kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Formulate the LPP.

SECTION –C

27. If the function $f: \boldsymbol{R} \to \boldsymbol{R}$ be defined by $f(x) = 2x - 3$ and $g: \boldsymbol{R} \to \boldsymbol{R}$ be defined by $g(x) = x^3 + 5$. Then find fog and show that fog is invertible. Also find $(fog)^{-1}$ and hence $(fog)^{-1}(9)$.

28. Verify Mean Value Theorem for the function $f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$.

29. Prove that: $2\tan^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5} = \frac{\pi}{2}$

30. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$.

31. Using properties of Determinants, Prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^2$$

32. If $y = \left[\log\left(x + \sqrt{1+x^2}\right)\right]^2$, Prove that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

SECTION –D

33. Show that the relation R defined by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$ on $A X A$ where

$A = \{1, 2, \ldots, 10\}$ is an equivalence relation. Hence, find equivalence class $[(3, 4)]$; $a, b, c, d \in A$.

34. Use the Product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve $x + 3z = 9$; $-x + 2y - 2z = 4$; $2x - 3y + 4z = -3$-

2z=4;2x-3y+4z=-3

35. s of tangents to the curve $y = x^3 + 2x - 4$ which are perpendicular to the line $x + 14y + 3 = 0$.

36. A toy company manufactures two types of dolls. A and B market tests and available Material downloaded from myCBSEguide.com. 16 / 16 resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit?

Sample Paper By Ms. Anu Priya